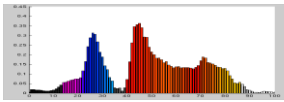


Dr. Qadri Hamarsheh

Signals and Systems
Lecture 2

Outline
- **Time-shifted signals.**
- **Continuous-Time signals using Matlab.**

Time-shifted signals

Suppose that $x(t)$ a C-T signal, the time-shifted version of $x(t)$:

- Shifted to the right by t_1 seconds (Delay), $x(t-t_1)$, t_1-positive real number.
- Shifted to the left by t_1 seconds (Advance), $x(t+t_1)$, t_1-positive real number.

✓ **Unit step function**

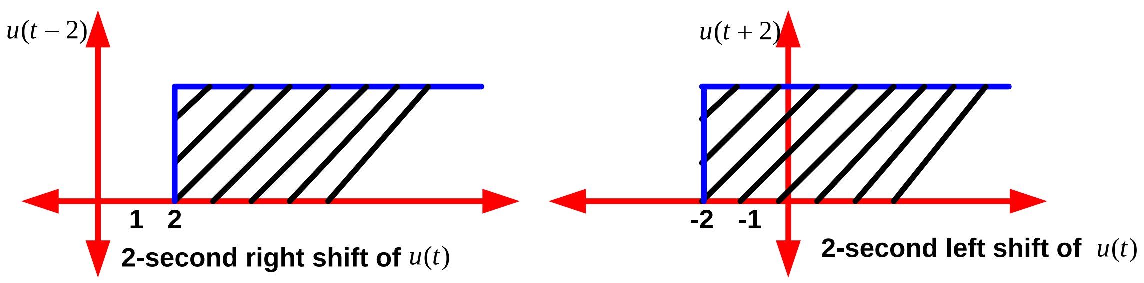


Figure 1-9

✓ **Impulse Unit**

$$\begin{cases} k\cdot\delta(t-t_1)=0, & t\neq t_1 \\ \int_{t_1-\varepsilon}^{t_1+\varepsilon} k\cdot\delta(\lambda-t_1)d\lambda = k, & \text{for any } \varepsilon>0 \end{cases}$$

any fixed positive or negative real number.

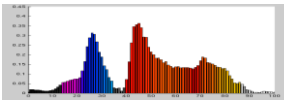
impulse with area k located at the point $t=t_1$.

time shift $k\cdot\delta(\lambda-t_1)$.

The time shifted unit impulse $\delta(t-t_1)$ is useful in defining the **sifting property** of the impulse given by

$$\int_{t_1-\varepsilon}^{t_1+\varepsilon} f(\lambda)\delta(\lambda-t_1)d\lambda = f(t_1),\ \text{for any } \varepsilon>0$$

Integrating the product of $f(t)$ and $\delta(t-t_0)$ returns a single number : the value of $f(t)$ at the location of the shifted delta function.



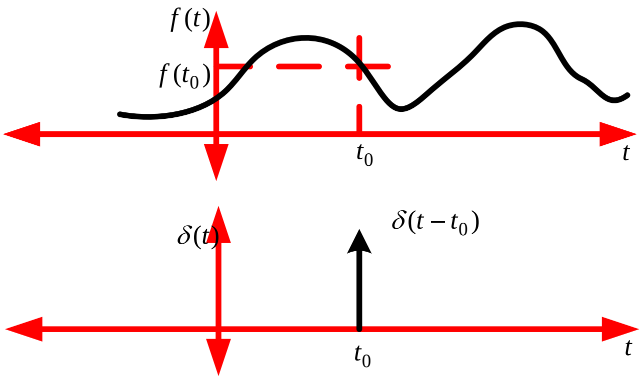


Figure 1-10

Steps for applying sifting property:

Examples:

1. $\int_0^{\infty} e^{-t}\cos(\pi t)\delta(t-4)dt$

Solution:

Step 1: find the variable of integration: t

Step 2: find the argument of $\delta(\bullet)$: $t-4$

Step 3: find the value of the variable of integration that causes the argument of $\delta(\bullet)$ to go to zero

$$t-4=0 \Rightarrow t=4.$$

Step 4: if the value in step 3 lies inside limits of integration, then take everything that is multiplying $\delta(\bullet)$ and evaluate it at the value found in step 3, otherwise "return" zero.

$t=4$ lies in $[0,\infty]$, so evaluate

$$e^{-4}\cos(4*\pi)=e^{-4}*1=e^{-4}$$

2. $\int_0^{\infty} t^3\delta(t+8)dt$

Solution:

Step 1: t

Step 2: $t+8$

Step 3: $t+8=0 \Rightarrow t=-8$.

Step 4: No, return 0.

3. $\int_0^{7} e^{-3t}\sin(6\pi t)\delta(3t-4)dt$

Solution:

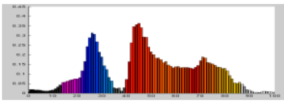
Step 0: change variables:

Let $\tau=3t \Rightarrow d\tau=3dt$

Integration limits: $t=0 \Rightarrow \tau=0 \quad t=7 \Rightarrow \tau=21$ then

$$\int_0^{21}(1/3)e^{-3\tau/3}\sin(6\pi\tau/3)\delta(\tau-4)d\tau$$

Step 1: find the variable of integration: τ



Step 2: find the argument of $\delta(\bullet)$: $\tau - 4$

Step 3: find the value of the variable of integration that causes the argument of $\delta(\bullet)$ to go to zero

$$\tau - 4 = 0 \Rightarrow \tau = 4.$$

Step 4: if the value in step 3 lies inside limits of integration, then take everything that is multiplying $\delta(\bullet)$ and evaluate it at the value found in step 3 , otherwise "return" zero.

$\tau = 4$ **Lies in** $[0, 21]$, **so evaluate** $(1/3)e^{-4}\sin(2\pi * 4) = 0$

An important application of the impulse signal is the decomposition of an arbitrary signal in terms of scaled and delayed impulses:

An arbitrary sequence $x(t)$ can be expressed as:

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t - \tau) dt$$

Continuous-Time signals using Matlab

We can use Matlab to plot the continuous-time signals using **linear interpolation** (approximated version) with suitable amount of samples according to the Nyquest theorem.

For example, to generate and plot the following signal

$$x(t) = e^{-0.2t} \cdot \sin(\frac{7\pi}{250} t), \quad 0 \le t \le 250$$

with 0.01 seconds increment for sampling process, the Matlab code for generation and plotting will be the following.

```
%Generation Steps of C-T Signal
%Generate the vector t for horizontal axis that contains
%the time values for which x values will be calculated, stored
%and plotted depending on the elements in the vector t.
t =0:0.01:250;
%Generate the first vector of the output containing
%the values of the expression exp(-0.2*t).
x1 = exp(-.02*t);
%Generate the second vector of the output containing
%the values of the expression sin((7*pi*t)/8).
x2 = sin((7*pi*t)/250);
%the resulting vector x must be multiplied element-by-element
%(multiplication of two output vectors), so we must
%use the dot before the multiplication operator.
x = x1.*x2;
% we can write x = exp(-.02*t).* sin((7*pi*t)/250);
%Plotting Step of C-T Signal
%plotting the Exponential Signal
subplot(3,1,1);
plot(t,x1,'r');
```

```
axis auto;
grid
xlabel('Time (Sec)');
ylabel ('Amplitude');
legend('Exponential Signal');
title('Plotting the C-T Signals using Matlab');
%plotting the Sinusoidal Signal
subplot(3,1,2);
plot(t,x2,'b');
axis auto;
grid
xlabel('Time (Sec)');
ylabel ('Amplitude');
legend('Sinusoidal Signal');
%plotting the Damped Exponential Signal
subplot(3,1,3);
plot(t,x,'g');
axis auto;
grid
xlabel('Time (Sec)');
ylabel ('Amplitude');
legend('Damped Exponential Signal');
```

Chapter1-1.m file

In figure 1-12 the intermediate different output signals are illustrated.

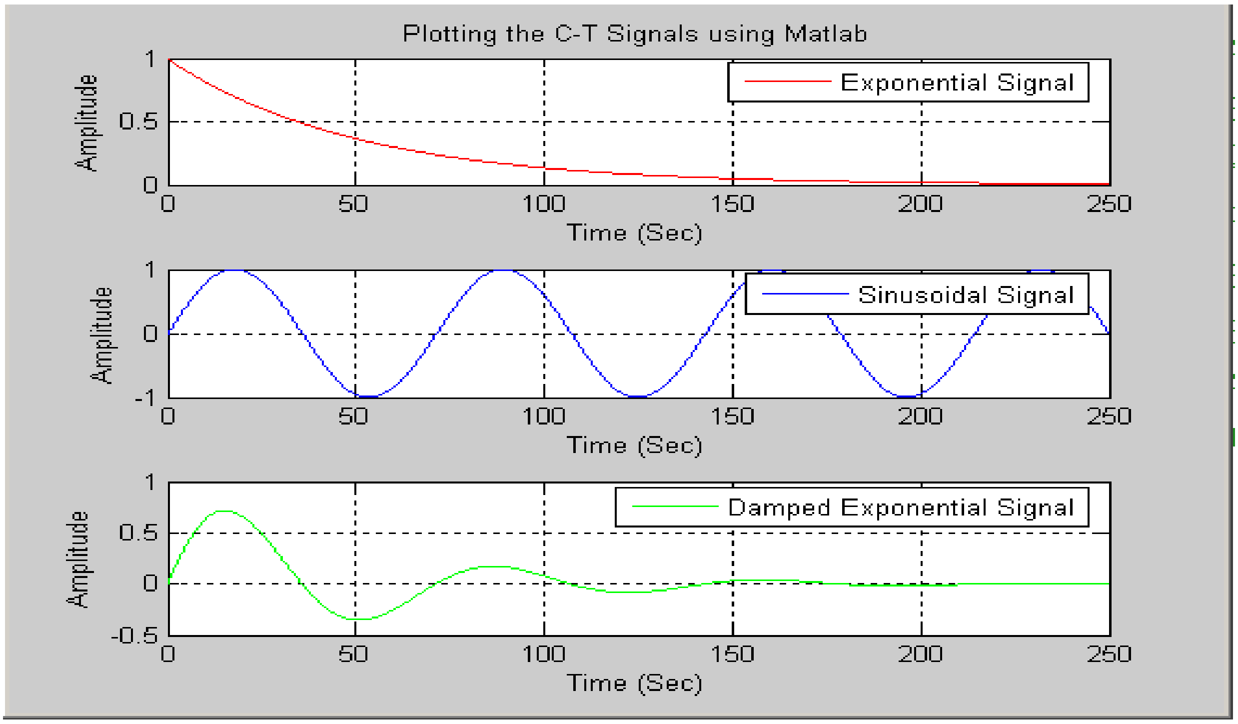


Figure 1-12